Improvement of argumentative competence by means of an ITS in math class at secondary school

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Abstract. Our aim in this paper is to show how students at secondary school level can improve their argumentative competence by means of an Intelligent Tutorial System (ITS) designed for learning geometry. After establishing the theoretical frame for our research, we compare the heuristic and discursive characteristics of some tutorial systems, including the one developed by our research team. Afterwards, we tackle the subject of complementarity between knowledge and competence in math class, and then we present an evaluation strategy of argumentative competence on the basis of relations within the subject-milieu system. Our study includes, particularly, structures of cognitive, semiotic and situational control associated to the development of argumentative competence in an Interactive Environment of Human Learning (IEHL)[1]. We also address the specificity of reference knowledge, the decontextualization of learning, the idea of mathematical proof, and the role of didactical agents.

Introduction

From the 1980's, most secondary school curricula were presented under the form of learning objectives, going beyond the tradition of contents and disciplinary methods. General objectives were then divided into specific, intermediate or terminal objectives, which should facilitate both teaching programming and learning evaluation. Even though this new approach provided the educational world with useful reference points over mathematical concepts, processes and attitudes, what usually stemmed from it was that learning consisted of a mere addition or juxtaposition of elements. The notion of competence appeared in this context. Defined as a know-how, competence takes place in situations of certain complexity; the degree of proficiency may progress during the school path and even beyond. Favouring competences in a math class contributes to establishing a different relation with traditional knowledge and to refocusing on the creation of thought and the development of autonomy.

Even though the development of a competence is a long-term job, situations of problem solving continue to be at the core of math learning. It is, furthermore, considered by the theory of didactic situations as the only way to "do math" (Brousseau, 1998). The aim is not to find a set of pertinent problems whose resolution presupposes satisfactory learning, nor to develop a set of situations diagnostically organized. In the first case it would mean to focus on situations-problems making reasonably significant the learning of a concept; in the second case, it would mean to foresee what difficulties the students will find and ask them to solve a related sub-problem in order to help them or orient them towards a series of adapted exercises. What we highlight in Brousseau's educational system modelling (1998) is that the teacher acts on the student's learning milieu (the teacher is the one choosing what problems are to be solved and what are the conditions of the problem solving) and that the student's interaction in this milieu provides information and feedback to all didactic agents (be it the teacher, a virtual pedagogic agent, a researcher, an evaluator or any other observer).

[1] Come from French expression «Environnement Informatique d'Apprentissage Humain (EIAH)» sanctioned by use.

In our research, the development of argumentative competences by means of an Intelligent Tutorial System (ITS) is not an objective in itself. We may think of some sort of "competential" tutor, made in the image of cognitive tutors, who propose situations-problems organized in a structured set of objectives, learning elements or pedagogic activities constituting a curriculum reference system. The improvement of the student's competence would depend on the achievements or difficulties found in relation to a programmed system. The development of a competence could, nevertheless, replace the learning objective, with the risk of subordinating mathematical concepts, processes and attitudes to the development of the competence-object. We rather consider that the evolution of a competence goes hand in hand with the progression of mathematical knowledge and that the tutorial system can act, just as a competent teacher, in response to the student's interactions in a milieu, in order to contribute to the creation of thought and the development of autonomy. When the tutorial system works within an Interactive Environment of Human Learning (IEHL), the improvement of argumentative competences can take place as long as the tutorial system admits cognitive control structures (in order to guarantee the consistent progression of knowledge), semiotic structures (to guarantee representation, treatment and communication), and situational structures (to guarantee devolvement and institutionalization).

In the following pages, the theoretical framework will allow us to compare heuristic and discursive characteristics of some tutorial systems. Afterwards, we will tackle the matter of complementarity between knowledge and competence, and then we propose an evaluation strategy of argumentative competences based on the tutorial system developed by our research team.

Construction of IEHL and mathematic didactics

When working on the design of a tutorial system, aimed at helping students in situation-problem solving through discursive interactions (messages), it is necessary to find an a priori way of analysis automatization. This means that, regarding particular conditions of a situation-problem and in a given didactic contract, the researcher-designer must at the same time find the totality of strategies mobilized by the student-solver and the totality of messages available to the tutor agent. This analysis job is frequent in mathematic didactics, but message management in response to student's interactions tends to be overlooked. A posteriori discursive exchanges are well dealt with, especially between a student and a teacher, as well as answering to the student's last action with the interface, but foreseeing authentic argumentative strategies seems like a lost battle.

On simulated social debate

From a broader perspective, all available IEHL aimed at teaching geometry at secondary school level don't consider simulated social debate in knowledge building. In problem solving, for instance, it is admitted that it is possible to generate a debate in a synchronic or asynchronic way through chat rooms, e-mails or forums. In these cases, knowledge construction and control are carried out in collaborative process of solution (among students) or through the intervention of a human tutor (generally, the current teacher). But if this kind of IEHL is useful to manage special situations where distance learning is required (hospitalized students, elite sport training, temporary family separation), its benefits are less obvious when it comes to regular learning. Because classmates or teachers are always present in order to kindle a possible debate, computer milieu becomes more of a tool than a mediation instrument for learning (Rabardel, 1995, Laborde, 2001a, b and Trouche, 2002). Personalized attention and debate animation in class are, nevertheless, more didactically costly. Not only the investment in time is a deterrent, but there is also the matter of pedagogic means to be mobilized in order to guarantee that all students work on the same situation-problem and that they respect its internal logic. Moreover, in some socio-cultural contexts, debate is regarded as suspicious. Let's consider, for instance, the break of social life and harmony laws (for example, in Quebec: Cornellier, 2003; in Japan : Sekiguchi & Miyazaki, 2000), or the feeling that a contradictory argument masks a personal attack. It is thus understandable why teaching practice resorts to it very occasionally.

Whereas traditional psychology points of view (v. g. Festinger, 1957, 1989; Vygotsky, 1978) give a central role to social debate in learning acquisition, other points of view, such as epistemological (Lakatos, 1984), semiotic (Duval, 1995) and situational (Brousseau, 1998) consider it necessary in math learning. By simulated social debate in a IEHL we mean an organized and directed discussion in which participants are a learning agent and a didactic agent. Contrary to instrumental learning activity or to a debate that may dilute the student's responsibility in the class (the tutorial system or the participant somehow manages a portion of knowledge at stake) the simulated social debate we recommend concedes a central role to the student. In other words, students are responsible of knowledge application, and debate progression depends essentially on their interactions with the interface, even during the period of deadlock management by the tutorial system (Cobo & al., 2005). We must point that, in a simulated social debate, didactic agents are particularly useful to support the management of meaning, representation, and respect of internal logic (Richard & al., 2003). When needed, the tutorial system allows to verify that symbols supported by the computing device have the same meaning for the student it is intended to have (semiotic control); that concepts in question refer in fact to aimed at models (cognitive control); and that the students work in the same situation-problem proposed to all (situational control).

About discourse

In order to simulate conversation and allow solution writing, our tutorial system uses a structural reasoning approach based on discursive expansion notions by Duval (1995) and graphic expansion by Richard (2004a). For Duval, reasoning is structured in reasoning steps – generically referred to as “inferences” for the purposes of this paper – in a logic process (eg. deductions, Aristotelian syllogisms) or dialogic process (eg. semantic or discursive inferences). According to Richard, these steps may mobilize several levels of semiotic representation (eg. figural inference) and regardless of the applied inference, the reasoning progresses by construction or accumulation of intermediate results (discursive consequences). In order to ensure the continuity of reasoning, inferences are linked together by proposition recycling or they are reinforced or opposed by their contents presenting a thematic coherence, in order to support or challenge the discussed thesis, as in Lakatos’ dialectic (1976). In the following pages, we will compare human reasoning and computing programming, showing its repercussions on the idea of mathematical model a priori; we will then introduce the notion of inferential graph in our system

Early tutorial systems showed formal expansion of reasoning, insisting on the functional structure of discursive propositions. Nevertheless, in addition to calculating processes (logic, arithmetic or algebraic), cognitive expansion is the privileged discursive expansion in math classes. A reasoning made through cognitive expansion requires the knowledge of definitions, rules or laws for a field of objects (Duval, 1995), without them needing to belong to any formal theory. In the eyes of a mathematical model, it is possible that the use of a definition or a property in order to justify an inference lacks global coherence, but the inference may be structurally coherent through the meaning of mobilized propositions. This phenomenon is common in geometry when reasoning is based on valid properties from a visual point of view, but false in the underlying geometric model. If we favour a mathematical model as frame of reference for computing programming, which presupposes the determination of a set of definitions and properties globally coherent in the model, we hinder the discovery of contradictions that could be part of a student’s strategy.

When we speak of cognitive geometry, it is because the point of comparison for our tutorial system is an inferential graph, formed by cognitive expansion. This graph allows certain common properties in a given didactic contract, such as inferential shortcuts – Richard’s macro properties (2004b) or Anderson’s macro operators (Koedinger & Anderson, 1993) – or any properties that remain locally coherent for the uncovering of contradictions. The structural approach to reasoning reconciles the difficulty of programming problem solving and the realization of proof using cognitive expansion. Firstly, this is so because the totality of inference rules may adapt to the logic of the problem and to the applied strategies in a given didactic contract. Secondly, because messages conveyed by the agent tutor allow simulating argumentation based on knowledge relating to the rules of inference. Thirdly, if the realization of a proof is necessary after a solving process, the graph is available for the drafting of a structured solution. Finally, the structural approach allows the system to consider control structures within the problem’s logic.

If there is no formal theory or cognitive theory – in the sense of a significant theory that interprets a formal theory – on which our system is based a priori, it engenders a model from the inferential graph and the algorithm (engine) that determines which messages are to be returned. The graph is constructed and modified by an expert (teacher, courseware designer), which means there is choice in the accepted hypotheses and rules of inference in a given problem. For example, the joint consideration of both true and false hypotheses in the context of a problem, where the accepted rules of inference are cognitively valid, admits the introduction of contradictions. If the student does not notice the contradiction, the algorithm may be placed in relation to the graph and generate an adapted message. In fact, if the structural approach allows for the tutor system to be equipped with a complete set of messages by default (e.g. El-Khoury & al., 2005), the expert must be able to adapt the content of each message when needed. In this way, when a contradiction appears – represented in the graph by an inference that does not lead to the conclusion – the messages may be reformulated and make the student be aware of the contradiction. When the student writes or chooses a proposition and the agent tutor returns a message, we may say that they collaborate in the resolution process. In other words, both are dealing with the same content and even the same inference. Nevertheless, according to their respective strategies, it is rather a cooperative process. The algorithm does not follow the student's progression in advance, but compares his/her actions with the graph. The student's actions and the tutor's messages constitute three layers of interactions that we developed in Cobo & al. (2005) and which refer to meaningful actions with the interface, the student's discourse and the mediation of the social milieu.

Related tutorial systems

Our research team works on two projects: AgentGeom (Cobo & al., 2005) and Turing (Richard & al., 2005). Apart from some minor differences on the research approach, the interface configuration and the use of models related to the programming exercise, these projects' goals are to build and study IEHL in order to improve competences in mathematics at secondary school level. Other tutorial systems are already devoted to supporting students by means of messages in problem solution, especially in problems involving proof. We summarize them in Table 1.

Table 1
Tutorial systems in geometry for secondary school level

	Underlying geometric model	**Tutor's intervention**
Baghera (Leibniz Laboratory, 2003)	Formal geometry	The student writes a solution, and a demonstrator (a deductive engine in first-order logic) verifies it. The returned file contains an annotated proof; each step of reasoning is followed by a annotation confirming whether it is correct or not. The file also contains a series of global annotations showing whether the proof is correct in relation to the problem's hypothesis.
Cabri-Euclide (Luengo, 2005)	Formal geometry	For each statement written by the student, the system first verifies whether the statement refers to an object constructed in the figure or whether the property is verified by an oracle of the Cabri-geometry. When the student uses a definition or a theorem, the system verifies the coherence of deductions or the constancy of proof, either automatically or at the student's request.
Geometry Explanation Tutor (Aleven & al., 2002)	Formal geometry	Based on the standards of cognitive tutors, the system considers a kind of divergence from the prescribed path in curricular architecture. When the student writes the justification of an inference, for instance, the system gives him/her a message according to a set of "right" or "partially right" explanations.
AgentGeom (Cobo & al., 2005) and **Turing** (Richard & al., 2005)	Cognitive geometry	Conceived following the particularity of a situation-problem and of a didactic contract, the system returns a message according to the nature of the ongoing process (solution or writing), the heuristic or discursive value of the student's action and the suggested level of control (see *Evaluation of Argumentative Competences*).

As in other related tutorial systems, *AgentGeom* and *Turing* use the construction of a geometric figure to generate the interactions between the student and the milieu, even if the situation-problem is not already modelled, in its statement, by a geometric figure. This point has to do with the matters of decontextualization, transfer or even acceptance of the contextual nature of the knowledge to be built and it presupposes that if properties of knowledge built by the student depend on his/her previous knowledge, they also depend on the characteristics of the milieu with which he/she interacts and the competences he/she may apply. On the other hand, geometric models underlying systems such as *Baghera*, *Cabri-Euclide* and *Geometry Explanation Tutor* are essentially formal models, and this determines the student's path and the tutor's intervention. Designing a tutorial system in cognitive geometry allows for the construction of an open architecture adapted not to a pre-existing geometric model but to a socio-cultural reality closely related to the particularity of a didactic contract. This means that in an inferential graph that is not necessarily formal, not only every inference may represent a cognitive, semiotic or situational state based on the a priori didactic analysis, but also the messages depending upon this allow to act in the interaction subject-milieu and, thus, in the evolution of mathematic competences. Furthermore, according to Guin (1996), the effectiveness of IEHL depends on the modelling of human behaviour that is, at the same time, previous and subsequent to the conception of the environment. Because the couple subject-milieu generates a model proper to it, the interactive device itself must allow an arrangement of heuristic and discursive characteristics (role of agents, programming of answers, strategies and sign systems available) according to the observed student's behaviour (Richard & al., 2005).

Knowledge and competence

Knowledge and competence are complementary. Knowledge useful to the exercise of a competence is the one constructed by an intellectually active student, and the extent of the competence depends directly on the pertinence and scope of the feeding knowledge. If knowledge is constituted by essential resources allowing to act adequately in a complex situation, the know-how proper to a competence presupposes the appropriation and intentional usage of notions and skills in question. The sequence for the development of a competence does not plainly go from simple to complex, or from parts to the whole, as in a plausible “competential” tutor. It is rather constructed according to multiple dimensions of a situation and of the intervention context. The starting point for the use and development of a competence is placed in the totality of the proposed challenge as well as the point of destination, that is, in a solution adapted to the initial problem. Thus, in an analogical way, we consider that math learning implies a spiral movement in which competences are useful to the acquisition of new knowledge that, in turn, cause the competences to evolve. Beyond this aspect of complementarity, let’s see how knowledge and competences are linked starting from a definition of conception where the subject and the milieu are defined mutually and didactically by the dynamics of their actions and reactions. In other words, the cognitive system is not the psychological subject, but the system subject-milieu taken as a whole. We then consider mathematical competences in three categories, in order to derive relations belonging to each argumentative competence.

In a model of knowledge for the reckoning of didactic situations and evaluation of conceptions made by the student, Balacheff and Margolinas (2005) show that the conception appears as a tool for the construction of a concept allowing the modelling of the student as learning agent, in relation with a didactic agent and a site where interaction and production take place. They consider that “conception is the equilibrium state of a system, and more specifically the equilibrium state of a loop of action/reaction of the subject-milieu system under proscriptive constraints of viability” (p.6). The term proscriptive refers to the necessity of conditions that guarantee the equilibrium of the system, which are not prescriptive because they don’t require a particular process in order to come back to the sought equilibrium. These proscriptive constraints do not refer to the way in which the equilibrium is reached, but to the criteria of that equilibrium, such as those related to structures of cognitive, semiotic and situational control associated to the development of an argumentative competence within an IEHL. Emphasizing the local nature of a conception, these authors characterize the conception by a defining set of problems (P) for which it brings resolution tools (R), based on representation systems (L) and a control structure (Σ) that allows for judgments and decisions. The conception C is formalized by the equality $C = (P, R, L, \Sigma)$ in such a way it defines the object of a conception by a class of equivalence of conceptions in relation with one of them. When a conception ($C\mu$) accounts for the text of mathematical knowledge, it defines an μ-object (mathematical object), that is, the class of equivalence of conception $C\mu$. Knowledge is, therefore, a set of conceptions having the same μ-object.

Table 2
Three categorizations of mathematical competence

MÉLS Training program in math at secondary school level	**TUNING** Training program in math at undergraduate university level	**PISA** Evaluation of competence and knowledge in math at secondary school level
• Solve a situation-problem (P) • Array mathematical reasoning (R, Σ) • Communicate by means of a mathematical language (L)	• Conceive mathematical proof (R, Σ) • Mathematically model a situation (L, T*) • Solve problems through mathematical techniques (P)	• Mathematical thinking and reasoning (R, Σ) • Mathematical argumentation (R, Σ) • Mathematical communication (L) • Modelling (L, T*) • Problem posing and solving (P, T*) • Representation (L) • Using symbolic, formal and technical language and operations (R) • Use of aids and tools (Σ)

There are many and very diverse models of competence. They are usually organized in general or transversal competences, as well as in disciplinary or transdisciplinary competences. They vary depending on the academic level to which they are destined (elementary vs. superior) and on the field of application (definition of a training program, evaluation of acquisitions among socio-cultural realities). We have chosen the following models of mathematical competence: *ministère de l’Éducation des loisirs et du sport du Québec* (MÉLS, 2003, 2005); European project

Tuning Educational Structures (TUNING, 2003); and international program OCDE/PISA (PISA, 2003), whose definition of competences is inspired on the works by Niss (1999, 2002) and his Danish colleagues. In Table 2, we summarize the relation between models of mathematical knowledge and competence. In T*, we gather two processes: transfer and decontextualization, that were not considered by Blacheff's and Margolinas' (2005) model, but we find them useful to compare the categorization of preceding competences. They belong to a larger process of mathematization (in the sense of PISA, 2003) and is related to control structures that go beyond purely mathematical field, both in terms of contents (situations of real life implying a transfer of academic acquisitions), and in terms of context (complex and dynamic environments of real life as well as reasoning tasks). On the other hand, because controls very often remain implicit and that in particular they bring together metaknowledge (Balacheff & Margolinas, 2005), we could have, by extension, included those processes in Σ.

The distinction between operators and controls is not absolute, but relative to a conception. In geometry, the operator may be the explicit and functional realization of a control, as the choice of a property in a pull-down menu or the writing of a definition in order to justify an inference. The explanation, given by the student, of mathematical definitions or mobilized properties in his/her reasoning allow him/her not only to guarantee a semiotic and cognitive control over the geometric figure, but also his/her actions guide the process of situational control when the simulated social debate takes place. Thus, in problem solving, when the tutorial system has already put down the heuristic and discursive characteristic of the milieu, the didactic agent must intervene relatively to operators and control structures that the system allows to explain in order to stimulate student's reasoning and act on argumentative competences. The improvement of these competences does not rest upon in the choice of one kind of problems, but in the quality of an argumentative process based on particular characteristics of each problem. It is even possible that the quality of an argumentation interferes with learning. For instance, if the didactic agent returned an oracle, this message would probably help the student in his/her reasoning or, at least, would encourage the argumentative process. But it could, at the same time, modify the target conception of the situation. On the other hand, if the student refused or failed to solve the problem, the tutor would have the responsibility to help the student in his/her path, just like any regular teacher, which would legitimate the occasional use of oracles. In order to reconsider the idea of proscriptive constraints of viability, we must understand that the notion of quality does not aim to the enrichment of argumentative method, but to the fact that messages from the didactic agent allow the tutorial system to guarantee that the learning agent works in the same situation-problem as that proposed and that he/she respects the internal logic – two necessary conditions for the development of autonomy.

Evaluation of argumentative competences

Our tutorial system is integrated to the didactic system that includes a regular teacher. In this perspective, the tutorial system is an adidactic milieu on which the teacher may act, modifying its heuristic and discursive characteristics. Nevertheless, the presence of a tutor agent in the system subject-milieu generates a didactic sub-system in which the tutor agent replaces momentarily the regular teacher. Simulating argumentation through messages, the milieu is thus no reduced to a place of production and interactions, because argumentative process itself acts on the conception of the subject, particularly in regard to operators and control. The evaluation of argumentative competences must, therefore, be carried out in two levels: in the real didactic relation, and in the simulated didactic relation. In the first case, the evaluation may result from comparing environments (for example, IEHL vs. paper-pencil) or milieus (for example, AgentGeom vs. Turing), as if the one responsible for the evolution of competences was the student's performance that the system can evaluate. In the second case, the evaluation must consider the progression of the student's argumentative know-how in a sequence of situations-problems, that may be put into perspective by some sort of closeness to an optimal result or according to other student's results, whose evolution criteria belong to the particularity of the subject-milieu system. Let us see some criteria in regard to our interactive device.

Generally speaking, the tutor agent is reactive. Apart from the usual greeting sentences, the tutor waits for a student's meaning action before sending a message. Messages depend on the current action, saved actions or the comparison with the elements of the inferential graph. Nevertheless, after a certain amount of time of no action, the system may decide to send a message and even "forget" an action already saved, as if the tutor was no longer able to follow the resolution process because of the elapsed time. Messages are classified according to the heuristic value of the action or according to the control levels (Cobo & al., 2005). Thus, when the messages are compared to the graph inferences, the suggestion is placed in one of these levels:

- 0 (or "situational") when the message is about the components of the didactic relation, that is, the milieu, the contract or the agents role;
- 1 (or "essentially semiotic") when it contains a descriptive mathematical suggestion, that may be related to P, R, L or Σ;
- 2 (or "essentially cognitive") when it is about the mathematical contents of the situation-problem (concepts and processes), among which there is the content relative to the target conception, which is the learning goal.

Thanks to markers that register the control levels used or called upon by the student, the improvement of argumentative competence considers the effect of the message within the developed strategy (path of the inferential graph starting from hypotheses of the situation and arriving to the conclusion) be it according to the coherence in inferences or the stability of reasoning. Moreover, when level markers remain mainly low, this shows not only that the student's autonomy is high, but also that the devolvement problem is linked more to effects of the didactic contract than to purely mathematical obstacles.

Conclusion

By choosing a structural approach in cognitive geometry, we were able to give form to a tutorial system that allows simultaneously reasoning, figure construction, argumentation and reckoning of interactions to understand and decide in an IEHL. If the modelling of knowledge and competences is necessary for the controlled construction of the system, it is also necessary to validate learning or to guarantee the creation of a competence. Our tutorial system does not aim to automatize the didactic relation, among other reasons because it is focused on the student, and because the improvement of argumentative competences needs, as a whole, the presence of human participants. Nevertheless, just in the image of the local nature of conception, that remains necessary for the acquisition of wider mathematical knowledge, the improvement of argumentative competences takes little steps in a sequence of complex situations-problems that take advantage of the mutual enrichment of conceptions. Tutor's messages and their effect on the student's reasoning also provide appreciation elements on the development of autonomy.

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