Pedagogical Support in the Solving and Proving of a Geometry Problem trough Interactions with an Agent Tutor

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Abstract: Our communication aims at showing how the pupil of the secondary school can appropriate geometrical abilities through interactions with an agent tutor in an artificial tutoring system. These abilities refer to the problem solving and the learning of the mathematical proof in geometry when the reasoning is expressed, in addition to by the action of the pupil to the interface of the system, by expansion of graphic propositions and discursive propositions. The system that we develop uses of the strategic messages of the tutor in an argumentative process that jointly considers the meaningful actions of the pupil, the meaning of the adopted propositions and the metacognitive reflection associated with the discovery of conjectures and the realization of proof. Finally, we present a short outline of a case study that shows the appropriation of geometrical abilities starting from a collection of interactions between the tutor and a pupil.

Introduction

The argumentation occupies a place of choice for the acquisition of knowledge in a class of mathematics. On the one hand, with its dialectical of the counterexample, Lakatos (1984) shows how the social debate and its discursive logic are in the heart of the mathematical discovery. In addition, the prospect constructivist, which relates to the change of design of the pupil and the going beyond of obstacles in the effective construction of knowledge, requires the reproduction of the constitutive characteristics of mathematical work:

> We know that the only means «of making» mathematics, it is to seek and solve certain specific problems and, on this subject, to posed new questions (Brousseau, 1998 : 61).

In this spirit, the exercise of dialectical of Lakatos of problems solving seems being essential for the acquisition of mathematical knowledge. However, according to Brousseau, the situations of training are to some extent paradoxical: it is considered that the pupil will have acquired the knowledge concerned only when it can implement it autonomously, without the support of the Master or of its companions. But if the pupil refuses, avoids or does not solve the problem, the Master is obliged to help him to advance. However, since the personalized attention and the animation of debates in class are didactically expensive (times to devote to it, means teaching to mobilize simultaneously to ensure, with each pupil, the devolution of the problem and the respect of the internal logic of the situation), one understands then why the teaching practice resort to it only occasionally. Moreover, one knows that the problems solving can also benefit from the contest of pars. That it acts of a proactive collaboration (when the par takes the initiative in the solving process) or reactivates (when the par discusses on what has thought or has been just made), it is often thanks to the intervention of a companion that the acquisition of a knowledge can take place.

Our proposal shows the application of a model of interactions that is based on research into didactic of mathematics for the programming of virtual pedagogical agents in an artificial tutoring system (that we call «AgentGeom» for the continuation), which gets to the pupils a metacognitive support to help them in their development of competences in problem solving and in mathematical reasoning. The agent architecture of AgentGeom is conceived as hybrid multiagent architecture made up of two distinct agents, the agent tutor corresponding to a reactive architecture (the tutor answers by a strategic message), the other with a deliberative architecture (which makes it possible to validate the decisions). The (human) pupil acts with the artifact system as a interface starting from the tools and of the buttons that are there, and the system answers according to the meaningful actions (in the broad sense of the term). The interactions which interest us are of discursive nature (exchange of messages expressed in the natural language or a symbolic language) or graph (exchange of messages expressed in the register of the geometrical figures). They fall under the constitution of a debate simulated with the interface of an interactive environment of human training which is intended for the problems solving in geometry and for their proof at pupils of the secondary school (Richard & al., 2003).

Relationships of AgentGeom in Current Research

The project that we presented has, like immediate antecedent, the Baghera project, directed by the researcher N. Balacheff within the Leibniz Laboratory (e.g. Leibniz Laboratory, 2003). In the same way that the intelligent tutorial system developed by this team of researchers, AgentGeom incorporates three principles, which we regard as being fundamental in the design of educational environments based on collaboration between human and artificial agents. Initially, this collaboration exceeds the paradigm dominating of the last decades. This one considered the computer as an autonomous device that conceived teaching only in its instructional function (Balacheff, 2000). Although we admit the existence of a knowledge of reference which underlies its learning (we speak then about epistemological validity), the pupil works with knowledge that it could exert and that is coherent with the situations with which it was face up to, this would be only partially coherent. Then, that leads to the fact that we regard education as the result of an emergent complex process that can emerge from interactions among agents having different and complementary abilities, and not be the result of the action of one isolated strategy or the accomplished goal of one isolated agent (Webber & al., 2002). Finally, just like Baghera, AgentGeom are conceived as a tutorial system multiagent of diagnosis that can identify knowledge of the pupils implemented starting from the interactions of those with the system. That comprises an adaptation of the system to the cognitive characteristics of the pupils and to the evolution of their mathematical knowledge.

However it has, among others, a fundamental difference between both projects. In AgentGeom, the checking of the accomplishment of the students' activities is carried out in real time. It is that we subscribe to the idea that the opened environment of learning could improve providing, in real time, support to the exploration processes, adapted to the individual necessities of each pupil (Bunt & Conati, 2002). We conceived our tutorial system so that it considers and controls instantaneously the set of the meaning actions of the pupil in the solving process. On the one hand, while identifying where he is in its discursive, cognitive or heuristic progress. In addition, by showing him, at its request or the own way of a human tutor, messages that direct its initiatives and collaborate in the fact that it is the pupil who obtains a solution (in opposition to any providential contribution in the solving process). This approach that proceeds by differentiated support for each pupil, according to the evolution of its solving process, carried out in a direct way, in real time and, which is adapted to the competence of the pupil, gives on our system a character of emergence, even of deterministic chaos (in a complex system). In the sense that the behaviour of a pupil in the solving process cannot be predicted by describing with precision the components of the system, or to be extrapolated by knowing what he uses like assumption in the process, although he arrives, in any event, with a solution.

Structure of the Interface, Structure of the Messages

The AgentGeom combines two basic functions of all education system. It is opened and it allows the attention to the diversity, in the sense that the teacher can choose between various types of geometrical problems that adapt to specificities of each pupil[1]. So that it is transferable to any environment, we conceived portable software developed on Web architecture. By using the most elementary resources that are available in this kind of environment, we make sure that the pupils can interact easily with the system starting from any current computer connected to a network. Consequently, it is a server software, i.e. the system tutor is on a server available for a group of pupils according to protocol HTTP. In addition, the agent system that we presented is personalizeable, since one of its parts can be managed by the regular (human) teacher. In this part, the teacher can create problems to solve, assign them to the pupils, examine the effective reasoning engaged by a pupil, modify the messages which are sent by the tutor to adapt them to discovered recurring strategies employed by pupils, etc.

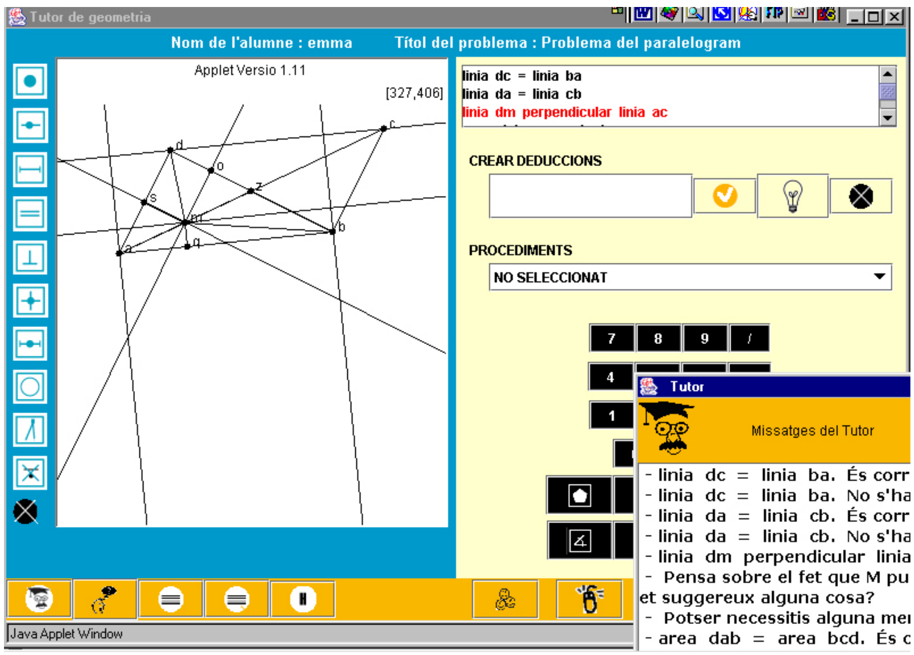

Figure 1: Aspect of the main interface of the pupil

In what follows, we show how the principal interface of the pupil arises (figure 1). On the left, there is a graphic area on which the pupil can build a geometrical figure by using primitives of construction (buttons on the left to place points or segments, to build two parallel or perpendicular straight lines, to define the intersection of two objects, etc.). The construction of a figure proceeds by forward chaining (o bottom-up chaining), heard as being a reasoning strategy, like the deductive reasoning, that starts with a set of rules (primitives of construction) and an initial state (geometrical objects drawn in the plan) and works forwards making inferences usually until some goal is reached (to build a complex geometrical figure). For example, when three points are connected with lines, the system automatically considers it as a triangle and it attaches to him the elementary geometrical properties of the triangles. In other words, if it is a triangle, it is that one can calculate his surface, to associate special lines to him, etc. On the right it is the area of deduction where the pupil can test, starting from a «editor of deductions», the exactitude of certain properties of the objects which it already built, like the measurement of an angle, a segment or the surface of a polygon, the comparison between two measurements (lower, higher or equal), the relations of parallelism or perpendicularity between straight lines, etc. AgentGeom is conceived so that a construction on the graphic area can be re-used by the deductive area in a unified geometrical model (graphic and textual). System checks if the properties

[1] When we speak, to the following section, of the didactic base of our approach, it is because we consider that to respect the pupil capacity means to recognize the mathematical validity in what he can do, while considering the inherent obstacles with the solving problem process and its devolution.

expressed in the deduction entered by the pupil are satisfied within the graphic area. These properties are calculated, as we said before, in real time, so there is no delay in the process of checking properties. For example, if one builds two straight lines parallel with the same third in the graphic part, the system regards these two constructions as assumptions for the deductive part. Thus, a pupil can deduce in the deductive part that these two lines are parallel between them by choosing the rule of suitable inference.

The construction of a figure element or a deduction determines the discursive-graphic units meaning (in the sense of Richard, 2004a, b) from which the reasoning of the pupil is constituted. The messages of the AgentGeom, which stick to the internal logic of the problem and the devolution of this one, react to a configuration of the meaning units preceding which reveal the strategy of the pupil in the solving process (search for a conjecture and proof). We reconsider the question of the strategy of the pupil within the following section.

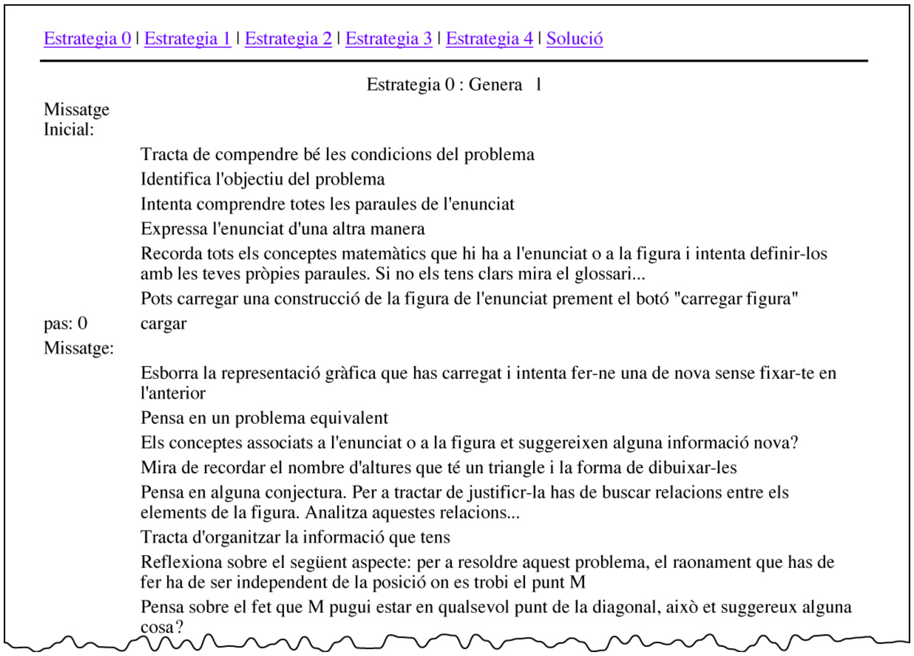

Figure 2: Example of the messages planned for the pedagogical agent according to the strategy of the pupil

The messages of the virtual pedagogical agent take as a starting point three models of interactions, that is to say the models of Cobo & Fortuny (2000), Kieran (2001) and Moschkovich (2004). The first model allows the evaluation of the progression of cognitive and heuristic competences in a collaborative solving process. The interactions of the model are compatible with the dialectical one of Lakatos, in the sense that they can relate to the formulation of the conjecture, the process of argumentation, the organization of knowledge or the mode of reasoning (e.g. treatment by congruence or equality of measurements). They are of «guided» type (e.g. to allow to check the effect of certain concepts or procedures which intervene), «alternate» (e.g. to propose to explore a case of figure already identified), «started again» (e.g. to avoid a dead end and to start again the discovery) and «co-operative» (e.g. to take stock of results obtained before a such working memory). The second model of interactions makes it possible to identify, in a solving process between two pars, if the cognitive and heuristic production is of the same order. That is, if the interlocutors reason on the same objects, contribute towards the formulation or the proof of the same conjecture and if their initiatives or their reactions are divided mutually in the argumentative process. The third model shows how a student learned from a tutor by participating in joint problem solving and appropriating aspects of mathematical practices. We give an example of the messages selected on figure 2. Messages are attached at each step defined into the strategy. Tutor tracks all actions performed by the pupil. Then he can localize where is the pupil within the resolution path. When tutor realizes that pupil is no longer following a valid path, a message is displayed. This is the one attached to the last step in the current strategy followed by pupil until that moment. It acts of a message associated at the step where the pupil is in its current strategy.

Didactic Base: Respect of the Pupil and Epistemological Validity

Contrary to the approaches which introduce an agent tutor to simulate the role of the teacher - with, possibly, an agent companion or a disturbing agent to simulate the attitude of pars -, we formulate the assumption that a virtual pedagogical hybrid agent is an autonomous being of nature compared to what should be a human model. In other words, the same virtual pedagogical agent can assume several traditional roles (tutor, companion or disturber, within the meaning of Aïmeur, 1998) while being defined in a distinct way compared to each one of these roles. This assumption is strong, but it makes it possible to ensure jointly the epistemological validity of the collaborative process solving while respecting the strategies of solving to which a pupil would have recourse in the particular context of a given problem. We must underline here that AgentGeom does not claim to replace regular teaching. By authorizing the solving of certain problems of geometry using a virtual pedagogical agent, we rather seek to supplement the situations of training which are likely to appear in an environment paper-pencil, but which is difficult to manage in situation of class or which could profit from the interactivity or the representation of the movement. This standpoint follows upon that which we engaged at the time of a realization of research on the remote training (Fortuny & al., 2002)[2].

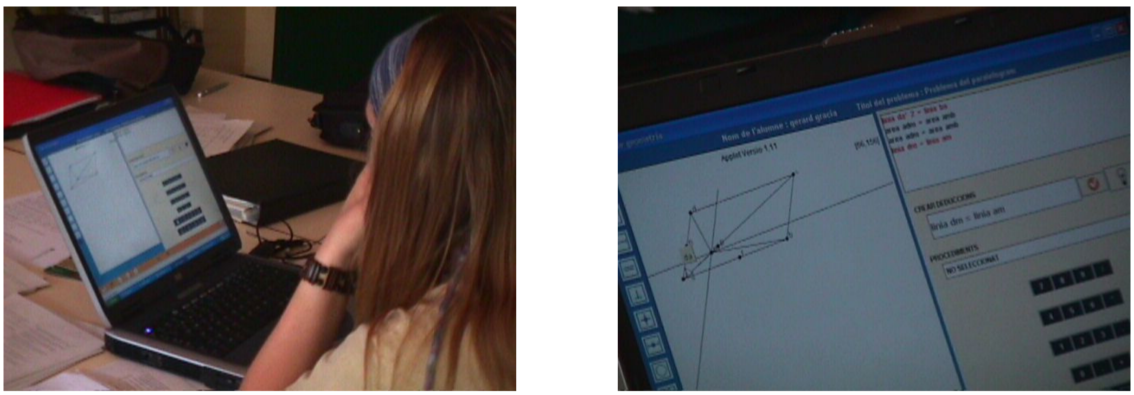

Figure 3: Pupil who is solving a problem with AgentGeom

We have obtained a complete system of messages to implement it in the AgentGeom combining different methodological strategies. On the one hand, we propose to the pupils to solve the problem in the environment paper-pencil by team of two in their asking to produce a written solution. The oral interactions are recorded in video then they are analyzed in connection with the solution written according to models' of Cobo & Fortuny (2000) and of Kieran (2001), in order to raise the discursive, cognitive characteristics and heuristics of the solving process. Later, we looked for the basic space of the problem (Cobo, 1998), that is to say, we identified all the possible forms to solve the problem and all the possible steps within each one of them. Next, we constructed the basic space of the human tutorial action. This one contains the set of the messages that relate to the procedures and concepts, the confirmation, reinforcement or invalidation to be given in the strategy chosen by the pupil, the encouragement in the continuation of a convenient strategy or the reorientation to be gotten in the event of blocking, in order to work out three networks form in the curriculum model that we used to constitute the role of the agents. Thus, the tutor can transmit the suitable messages at the time of any intermediate step of the solving process, as would do it a human tutor. Finally, after a first structuring of the messages in the AgentGeom, we adjust the suitability of the messages when the pupil, taken individually, solves the problem with the interface of the data-processing device (figure 3).

Appropriating Geometrical Proof Abilities: Cases Study though Interaction with a Agent Tutor

We will say that the study that we are conducting seeks to discover which geometrical proof abilities students can acquire and how they can be actively transformed after the suitable adjustment of the messages that we have mentioned above. The observation retains three layers of actions/interactions: the *meaningful actions* of the student

[2] One could also consult the interactive site <http://www.edu365.com/aulanet/intermates/index.htm>.

with the interface, recorded inter alia by a digital camera, the *discourse of the student*, expressed by discursive-graphic units, and the *mediation of the «social milieu»* with the artificial tutor. These layers of actions/interactions are connected to one another, i.e., the information that is in a layer is necessary to interpret the information that is in another layer. For example, the contents of the meaningful actions has to be explained in the light of the interaction with the agent tutor, since this one is mediated by the simulated social interactions; the meaningful actions have to be explained compared to the discourse of the student, since it reveals the strategy of the student and the goal of his or her actions.

The AgentGeom was not conceived in the spirit of a deductive reasoning training, which proceeds by posing a problem and, at the same time, requires to connect propositions by formal expansion in order to show a conclusion proposition from the hypothesis of the problem, as did the first systems tutors intended for learning geometry (e.g., Geometry Proof Tutor[3]). Not because the AgentGeom does not support this approach, but because it is of nature much more extensive. The pupil must be able to organize his knowledge to construct figures then act on these in order to discover and validate a reasonably convincing conjecture. This comes as a result of social consensus and is based on relevant knowledge that the student can communicate to the interface. For this reason, the proof abilities that the student can appropriate with the AgentGeom are primarily of cognitive type.

However, there is a difference between the agent mediator geometrical model and the geometrical knowledge that the pupil can apply. This is because the geometrical model of the system is coherent with its internal representation possibilities (semiotic and cognitive). While the student's geometrical knowledge can be coherent with respect to a given situation, nevertheless, he or she can be missing global coherence vis-à-vis what the geometrical model of the system is supposed to represent. In fact:

> The machine-generated construction is often different from the one asked of the student (who usually is working in a cognitive geometry; i.e., within a certain context of definitions, theorems, and exercises which do not include all of geometry) and is useful as a metaphor relating Euclidean geometry to analytic geometry and linear algebra (Allen & Trilling, 1997, p. 196).

The definition of the basic space of the problem thus begins to mitigate the effect of an undesired cognitive difference between the pupil and the system. Functionally, we define the proof as being the synthetic unity of the pupil's discourse directed towards the validation of the conjecture in which the agent tutor intervenes cognitively as social mediator by comparison with the basic space of the problem. At the end of proof, we consider that the student acquired a geometrical knowledge when there is a cognitive profit at the time of a state of balance between the student and the social milieu, even if this profit moves away from the principle aim of the problem.

Conclusion

In summary we can highlight, on the one hand, the aspects related to the technological characteristics of the AgentGeom, on the other, its pedagogical application when helping students in the problem solving process. Regarding the technological characteristics, the AgentGeom is a tutorial multi-agent system that combines two basic functions in any educational system: it is open and allows the attention to the diversity, as it gives the necessary mechanisms so that the teacher can: a) broaden the series of problems; b) manage the system; c) creating news problems; d) assign them to his/her pupils according to their cognitive characteristics; e) examine the effectiveness of their solving processes; and finally f) modify the system of messages that she can send to each pupil in each problem according to the strategies that have been chosen. Moreover, the AgenGeom has the characteristic of checking out and verifying in an instantaneously all the actions carried out by the pupils.

Regarding its pedagogic collaboration with pupils in the resolution of problems, we can say that the AgentGeom has generated changes in the pupils' learning. In our research, these changes have been related to appropriations, on the part of the pupils, of the form of producing inferential sentences and understanding both the meaning and the need of the mathematical proofs in the resolution of geometry problems. Furthermore, these changes have been evident through transformations in the performances of the pupils who have evolved from the graphical and inferential areas of the AgentGeom towards a use of the argumentative process based on a shared use

[3] Advanced Computer Tutoring, Inc. (1989); 4516 Henry Street; Pittsburg, PA 15213; version 1.1.

References

Aïmeur, E. (1998). Application and Assessment of Cognitive Dissonance Theory in The Learning Process. *Journal of Universal Computer Science, 4*(3), 216-247.

Allen, R. J., & Trilling, L. (1997). Dynamic Geometry and Declarative Geometric Programming. In J. R. King & D. Schattschneider (Ed.), *Geometry Turned On. Dynamic software in learning, teaching, and research* (pp. 193-197). Washington: MAA Notes Series.

Balacheff N. (2000) Teaching, an emergent property of eLearning environments. In *Proceedings of the International Conference on Intelligent Tutoring Systems 2000*, Nice. Retreived at http://www-didactique.imag.fr/Balacheff/TextesDivers/IST2000.html.

Brousseau, G. (1998). *Théorie des situations didactiques*. Grenoble: La Pensée Sauvage.

Bunt A and Conati C. (2002). Assessing Effective Exploration in Open Learning Environments Using Bayesian Networks. In *Proceedings of the International Conference on Intelligent Tutoring Systems 2002*, Biarritz. Retreived at http://www.cs.ubc.ca/%7Econati/my-papers/its2002BuntConati.pdf.

Cobo, P. & Fortuny, J. M. (2000). Social interactions and cognitive effects in contexts of area-comparison problem solving. *Educational Studies in Mathematics, 42*, 115-140.

Cobo, P. (1998). *Análisis de los procesos cognitivos y de las interacciones sociales entre alumnos (16-17) en la resolución de problemas que comparan áreas de superficies planas. Un estudio de casos.* Doctoral dissertation. Universitat Autònoma de Barcelona.

Fortuny, J. M., Giménez, J. & Richard, P. R. (2002). Distance Education at Secondary Levels : Contexts and Norms for the Learning of Mathematics. *Proceedings of the E-Learn 2002 World Conference of Association for the Advancement of Computing in Education*.

Kieran, C. (2001). The Mathematical Discourse of 13-year-old Partnered Problem Solving and Its Relation to the Mathematics that Emerges. *Educational Studies in Mathematics, 42*, 115-140.

Laboratoire Leibniz (2003). Baghera Assessment Project: Designing an hybrid and emergent educational society. In Soury-Lavergne S. (ed.), *Rapport pour la commission européenne, Programme IST, Les Cahiers du Laboratoire Leibniz nº 81*, Grenoble.

Lakatos, I. (1984). *Preuves et réfutations. Essai sur la logique de la découverte mathématique*. Paris: Hermann.

Moschkovich, J. N. (2004). Appropriating mathematical practices: a case study of learning to use and explore functions through interaction with a tutor, *Educational Studies in Mathematics, 55*, 49-80.

Richard, P. R. (2004a). *Modélisation du comportement en situation de validation*. Berne: Peter Lang.

Richard, P. R. (2004b). L'inférence figurale: un pas de raisonnement discursivo-graphique. *Educational Studies in Mathematics, 57* (2). In press.

Richard, P. R., Fortuny, J. M., Cobo, P. & Aïmeur, E. (2003). Stratégie argumentative et système tutoriel pour l'apprentissage interactif de la géométrie. In *Actes de l'EMF-2003 (Espace mathématique francophone)*, Tozeur.

Webber, C., Bergia, L., Pesty, S., Balacheff, N. (2002). The Baghera project: a multi-agent architecture for human learning. In *Proceedings of the Workshop Multi-Agent Architectures for Distributed Learning Environments (AIED2001)*, 12-17, San Antonio.

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